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On the base space of a semi-universal deformation of rational quadruple points.
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To compute the base space of the semiuniversal deformation of an (isolated) singularity is a notoriously hard problem. Very few nontrivial examples are known, and of the published computations a considerable number contain mistakes. In the last few years a number of new techniques have been developed that allow one to deal with semiuniversal deformations of, e.g., rational surface singularities more successfully than before. The paper under review is the result of one of the most striking of these advances.

In the course of the past decade, mainly in the Dutch singularity school, a deformation theory for nonisolated hypersurface singularities has been constructed. Roughly, the idea is as follows: one deforms the singular space and its singular set—the latter equipped with the correct analytic structure—and keeps the nearby fibres of the singular set in the singular locus of the nearby fibres of the singularity. These admisssible deformations define a finite-dimensional deformation theory that admits semiuniversal deformations. Things are particularly simple if the singularity at each generic point of the singular curve is of transversal type A_1. Then the right structure of the singular locus is reduced.

In the surface case there is a close connection with deformations of normal (hence isolated) singularities. The authors have proved elsewhere [Math. Ann. **288** (1990), no. 3, 527–547; MR1079877 (92d:32050)] that simultaneous resolution of the fibres of an admissible deformation of a weakly normal two-dimensional hypersurface singularity gives a deformation of its normalization, and that the semiuniversal base spaces of the hypersurface singularity and of its normalization agree up to a smooth factor.

In the paper under review the authors apply their technique to the case of rational quadruple points. They determine the singular locus of a generic projection of such a singularity into complex 3-space. The ideal I of this space curve together with its primitive ideal $\int I$ is the central object for all that follows. For $f \in \int I$ the hypersurface $\{f = 0\}$ is singular along the curve. If f and g in $\int I$ are I^2-equivalent ($f - g \in I^2$), then the (admissible) semiuniversal base spaces of the hypersurfaces differ only by a smooth factor. In this way the authors show that there is a series of rational quadruple points, the n-stars ($n \in \mathbf{N}$) such that each projection of a rational quadruple point is I^2-equivalent to a generic projection of exactly one n-star.

They determine the versal base spaces of the n-stars through a tedious calculation of the semiuniversal admissible deformations of their generic projections. The result is a series of spaces, $B(n)$, that make up all versal base spaces of rational quadruple points up to smooth factors. $B(n)$ has $n + 1$ irreducible components, of pairwise distinct dimensions. Only the biggest and the smallest components are smooth, but all components have smooth normalizations.

Reviewed by *Kurt Behnke*